Exact solutions for equilibrium configurations of charged conducting liquid jets

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A wide class of exact solutions is obtained for the problem of finding the equilibrium configurations of charged jets of a conducting liquid; these configurations correspond to the finite-amplitude azimuthal deformations of the surface of a round jet. A critical value of the linear electric charge density is determined, for which the jet surface becomes self-intersecting, and the jet splits into two. It exceeds the density value required for the excitation of the linear azimuthal instability of the round jet. Hence, there exists a range of linear charge-density values, where our solutions may be stable with respect to small azimuthal perturbations.

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I. INTRODUCTION

Cylindrical jets are known to be unstable with respect to small surface perturbations because of the development of the Rayleigh instability caused by capillary effects [1]. For electrically charged jets, electrostatic forces are an additional factor that determines the system behavior. The Coulomb interaction of electric charges suppresses the large-scale axial capillary instability. On the contrary, it can lead to the growth of nonaxisymmetric modes of disturbances which are stable for the uncharged jet (see [2–4] and references therein). In order to understand the main laws governing the behavior of a charged jet, it is important to define conditions when the mutual compensation of the electrostatic and capillary forces is possible, as well as the conditions when such compensation is impossible. Therefore, the necessity arises to determine the region of existence of stable solutions for the problem of the equilibrium configurations of the jet surface.

In this paper, we consider possible equilibrium shapes of a charged infinite jet of a conducting liquid (an electric charge distributes itself over the liquid surface so that the electric field potential is constant everywhere inside of the conductor). In doing so, we restrict our analysis to the particular case of the azimuthal deformations of the initially circular jet (axial deformations can be suppressed by the longitudinal magnetic field). For this case, the instability is induced by the electrostatic forces, while the surface tension forces play a stabilizing role.

We are presently only aware of a few nontrivial solutions of the classical problem in electrostatics, that is, the problem of finding the stationary configurations of the charged surface of a conducting liquid (the flat surface, the circular cylinder surface, and the sphere belong to the trivial solutions). We should primarily mention the so-called Taylor cone. In Ref. [5], Taylor has demonstrated that the surface electrostatic pressure for a cone with angle 98.6° is inversely proportional to the distance from its axis and, hence, can be counterbalanced by the capillary pressure (the sole exception is the cone apex where the force balance condition is violated). Recently, the first author of the present paper has shown [6] that for the case of plane symmetry, the problem of the steady-state shape of the free surface of a conducting liquid in an external electrical field is mathematically similar to the problem of the progressive capillary wave solved by Crapper [7]. The analogy provides an easy way of solving the electrostatics problem [8]. Finally, the method of constructing exact solutions for the equilibrium configurations of the charged two-dimensional drops was proposed in Ref. [9]. It should be noted that apart from the transformations, this problem coincides with that of the stationary shape of a two-dimensional air bubble in a circulatory ambient flow. The solutions found by Zubarev for an arbitrary mode number n [9] were independently given by Crowdy for $n=2$ [10] and by Wegmann and Crowdy for $n=3,4,5\ldots$ [11] in the problem with the bubbles. The approaches developed in Refs. [9–11] turn out to be useful for the following analysis of possible configurations of charged jets.

The article is made up as follows. In Sec. II, we give the equations defining the equilibrium configuration of the charged surface of a conducting liquid for the case of plane symmetry. It is shown that conformal transformations allow us to reduce the investigation to the analysis of a nonlinear boundary-value problem on a half-plane for the Laplace equation. In Sec. III, using the results of Refs. [9–11], we obtain exact solutions for the jet configurations corresponding to the azimuthal mode numbers $n=2,3,4\ldots$. In Sec. IV, the equilibrium surfaces corresponding to our solutions are investigated. We formulate the conditions under which the surfaces become self-intersecting, and the jet splits into several separate jets. In Sec. V, we analyze the dependence of the jet surface deformation amplitude on the control parameter (linear electric charge density) for different azimuthal numbers. It turns out that we deal with a soft loss of stability of the round jet surface for $n=2,3,4$ (supercritical bifurcation) and with a hard loss of stability for $n>4$ (subcritical bifurcation). Section VI contains our conclusions and some remarks concerning conditions whereby the solutions obtained can play an important role in the jet behavior.

II. INITIAL EQUATIONS

Let us write the equations of electrostatics that describe a stationary profile of the charged surface of the conducting

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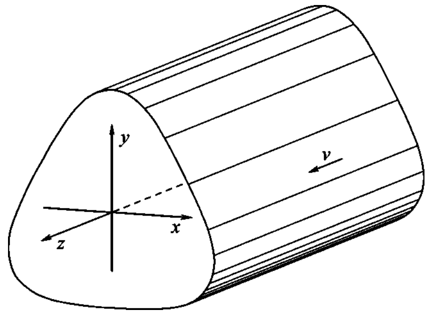

FIG. 1. The geometry of the azimuthally perturbed liquid jet is represented schematically. The jet cross section is constant along the z axis.

liquid jet with constant cross section along the direction of its motion (see Fig. 1). The distribution of the electric-field potential φ in the plane of the jet cross section $\{x,y\}$ is determined by the Laplace equation,

$$\varphi_{xx} + \varphi_{yy} = 0.$$

It should be solved together with the condition that the conductor surface is equipotential,

$$\varphi = 0,$$

and also the condition that the field of the charged conductor coincides at infinity with the field of a uniformly charged straight filament,

$$\varphi \to - Q \ln(x^2 + y^2), \quad x^2 + y^2 \to \infty, \tag{1}$$

where Q is the linear electric charge density of the conductor.

The jet will be considered to move at the constant velocity v along the z axis of the Cartesian coordinate system (a fluid is at rest in the system of coordinates moving with the jet). Then the equilibrium relief of the charged boundary of a conducting liquid is determined by the Laplace-Young stress condition, that is, the balance condition for the electrostatic and capillary forces acting on the surface,

$$(8\pi)^{-1}(\nabla \varphi)^2_{\varphi=0} + \alpha C + p = 0, \quad \varphi = 0, \tag{2}$$

where α is the surface tension coefficient and C is the local curvature of the surface. The constant p is expressed in terms of the jet velocity (v), liquid density (ρ), and the external (p_e) and internal (p_i) pressures,

$$p = p_i - p_e - \rho v^2/2.$$

It is apparent that an infinitely long cylindrical jet with a circular cross section (i.e., a round jet) gives the trivial solution of the problem. In what follows, we will obtain its nontrivial solutions corresponding to the azimuthal deformations of the round jet surface.

For convenience, we convert to the dimensionless variables

$$x \to Q^2(2\pi\alpha)^{-1}x, \quad y \to Q^2(2\pi\alpha)^{-1}y,$$

$$\varphi \to 2Q\varphi, \quad p \to 2\pi\alpha^2 Q^{-2} p.$$

By analogy with Ref. [7], we choose $f=\ln|\nabla\varphi|$ as a new unknown function, and the pair of conjugate harmonic functions φ and ψ as new independent variables (the condition ψ=const defines the electric field lines). The so-called complex potential $w=\varphi - i\psi$ is an analytic function of the complex variable $z=x+iy$. The complex expression $\ln(-dw/dz)$ is also an analytic function, and, as a consequence, the real functions

$$f \equiv \mathrm{Re}\,\ln(-\,dw/dz) = \ln|\,\nabla\,\varphi|, \tag{3}$$

$$\theta \equiv -\,\mathrm{Im}\,\ln(-\,dw/dz) = \arctan(\varphi_y/\varphi_x) \tag{4}$$

are conjugate harmonic functions of the variables φ and ψ. In particular, this implies that the function f satisfies the Laplace equation,

$$f_{\varphi\varphi} + f_{\psi\psi} = 0. \tag{5}$$

The boundary conditions for f can be derived from the expressions (1) and (2). We get

$$f_\varphi = pe^{-f} + e^f, \quad \varphi = 0, \tag{6}$$

$$f \to \varphi, \quad \varphi \to -\infty, \tag{7}$$

where we have taken into account that the fluid surface curvature can be expressed in terms of the functions f and φ in the following way: $C=-(f_\varphi \exp f)_{\varphi=0}$. Since in the limit $|z| \to\infty$ we have $w\to -\ln z$ for the complex potential and, consequently, a closed surface corresponds to changing ψ by 2π, we add the condition for periodicity of f with respect to the variable ψ,

$$f(\varphi,\psi) = f(\varphi,\psi + 2\pi), \tag{8}$$

closing the system of equations for the function f.

Thus, the problem of finding the steady-state profile of a cylindrical jet surface amounts to studying the boundary-value problem (5)–(8) on the half-plane $\varphi \leqslant 0$.

III. EXACT SOLUTIONS

A wide class of particular solutions of Eqs. (5)–(8) was obtained in Ref. [9]. They are given by the formula

$$f = \ln\left(\frac{l-1}{2}\right) + \ln\left(\frac{1 + a^2b^2e^{2n\varphi} + 2abe^{n\varphi}\cos(n\psi)}{a^2 + b^2e^{2n\varphi} - 2abe^{n\varphi}\cos(n\psi)}\right) + \varphi, \tag{9}$$

where we put

$$a = \sqrt{(n-1)/(n+1)}, \quad b = \sqrt{(n-l)/(n+l)}.$$

In these expressions n is the azimuthal mode number ($n=2,3,4,\ldots$) and $l=\sqrt{1-4p}$ is the parameter characterizing the amplitude of the surface deformation of the round jet.

Let us construct the equilibrium surfaces corresponding to the solution (9). It follows from the definitions (3) and (4) that the inverse transformation from the variables φ and ψ to x and y is determined by the relation

$$z = -\int \exp(-f + i\theta)dw, \tag{10}$$

where θ is the angle of inclination of the electric field intensity vector to the abscissa direction. Taking into account that f and θ are conjugate harmonic functions and thus the Cauchy-Riemann conditions are satisfied ($\theta_\psi = f_\varphi$ and $\theta_\varphi = -f_\psi$), we obtain from Eq. (9)

$$f - i\theta = \ln\left(\frac{l-1}{2i}\right) + 2\ln\left(\frac{1 + abe^{nw}}{a - be^{nw}}\right) + w.$$

Substituting this expression into Eq. (10), we get

$$z = \frac{2i}{(l-1)}\int \left(\frac{at^n - b}{t^n + ab}\right)^2 dt = \frac{2ia^2 t(t^n + b/a^3)}{(l-1)(t^n + ab)}, \tag{11}$$

where we have introduced the notation $t = \exp(-w)$. The transformation corresponding to Eq. (11) maps the unit circle $|t| = 1$ onto the free surface of a liquid. The algebraic expression (11) relates the results of Ref. [9] and the results of Refs. [10,11], where the conformal mapping to the exterior of the unit disk was applied for the problem related to an analysis of the profile of a two-dimensional bubble in a circulatory ambient flow.

Bearing in mind that $\varphi = 0$ at the boundary and, consequently, $t = \exp(i\psi)$, we obtain from Eq. (11) that the sought-for equilibrium surfaces are given by the following parametric expression:

$$z = \frac{2ia^2 e^{i\psi}(e^{in\psi} + b/a^3)}{(l-1)(e^{in\psi} + ab)}, \tag{12}$$

where ψ plays the role of the parameter. The closed surface corresponds to the change in ψ in the range $0 \leqslant \psi < 2\pi$.

Let us return to the real variables. Having separated the real part from the imaginary one in Eq. (12), we find

$$y = \frac{a^{-1}b\cos(n\psi - \psi) + a^3 b\cos(n\psi + \psi) + (a^2 + b^2)\cos\psi}{[1 + a^2b^2 + 2ab\cos(n\psi)](l-1)/2}, \tag{13}$$

$$x = \frac{a^{-1}b\sin(n\psi - \psi) - a^3 b\sin(n\psi + \psi) - (a^2 + b^2)\sin\psi}{[1 + a^2b^2 + 2ab\cos(n\psi)](l-1)/2}. \tag{14}$$

The formulas (13) and (14) represent a family of exact two-parametric solutions for the equilibrium shape of a charged jet of a conducting liquid. To the best of our knowledge, these solutions for the jet configurations have not been considered so far.

The particular solutions (13) and (14) of the initial equations relate to the case $p \neq 0$. Note that for $p = 0$ it is possible to find the general solution of Eqs. (5)–(8). It has the following form:

$$f = \ln(1 - c^2) - \ln(c^2 - 2ce^{-\varphi}\cos\psi + e^{-2\varphi}) - \varphi, \tag{15}$$

where the constant c satisfies the inequality $0 \leqslant c < 1$. This solution is 2π-periodical with respect to ψ, so that it corresponds to the azimuthal number $n = 1$. The equilibrium surface is given by the parametric expression

$$z = \frac{1}{1 - c^2}(e^{i\psi} - c^2 e^{-i\psi} - 2ci\psi).$$

One can readily see that this surface has the self-intersections for arbitrary c. Thus, the solution (15) with the mode number $n = 1$ is physically meaningless.

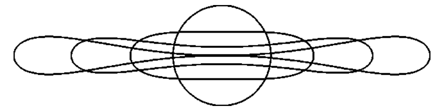

FIG. 2. Typical (superposed) cross sections of the charged jet of a conducting liquid for $n=2$ and parameter values $l = 1.86, 1.94, 1.88, 2$. The cross-section areas are normalized to a constant.

IV. CONDITIONS OF THE JET SPLITTING

The solutions (13) and (14) obtained in the previous section enable us to find exact critical values of the linear charge densities required (i) for the onset of the azimuthal instability of the round jet and (ii) for the jet splitting.

In the limit $l \to n$, expressions (13) and (14) for the equilibrium shape of a charged jet of a conducting liquid define circles of radius $2/(n+1)$, which correspond to the unperturbed state of the jet, namely, to the round jet. With decreasing the parameter l, the jet surface is deformed. At certain n-dependent critical values l_n of the parameter l, the region occupied by the liquid ceases to be simply connected, and the jet splits (see Figs. 2–4). For $1 < l < l_n$, the solutions are physically meaningless so that for fixed n the set of the problem solutions corresponds to the interval $l_n \leqslant l \leqslant n$.

We now determine the critical values of the parameter l. For $n = 2$, the condition of the surface self-intersection has the form

$$x = 0, \quad \psi = \pi/2,$$

which corresponds to the singularity at the point $x = y = 0$. It follows from this condition that l_2 is a root of the quadratic equation

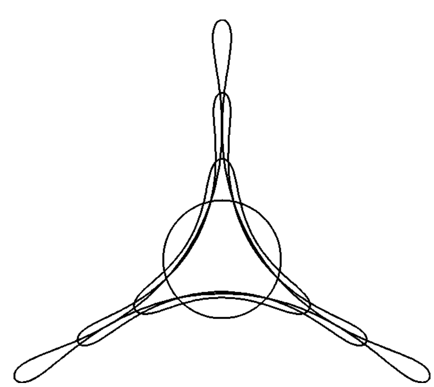

FIG. 3. The cross sections of the jet for $n=3$ and parameter values $l = 2.53, 2.59, 2.78, 3$.

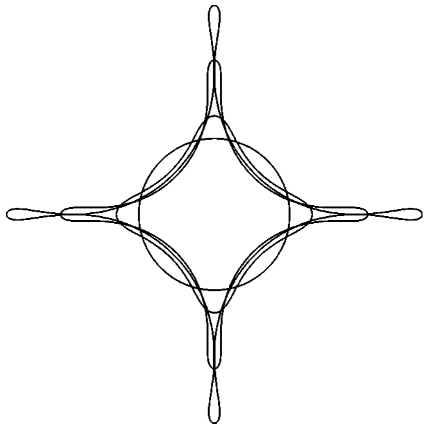

FIG. 4. The cross sections of the jet for $n=4$ and parameter values $l=3.19, 3.35, 3.8, 4$.

$$7l_2^2 - 6l_2 - 13 = 0.$$

Only one solution of the equation, $l_2 = 13/7 \approx 1.86$, meets the requirement $1 < l_2 < 2$. The jet splits into two equal parts at this value of the parameter l (see Fig. 2).

For $n>2$, the condition of the self-intersection of the surface can be written as

$$x = 0, \quad dx/d\psi = 0.$$

Hence it follows that the critical values of l, i.e., the quantities l_n, are determined by the set of equations

$$a(n+1)(a^2+b^2)\cos\psi = 2b(n-1)[n\sin(n\psi)\sin\psi + \cos(n\psi)\cos\psi], \quad (16)$$

$$a(n+1)^2(a^2+b^2)\sin\psi = 2b[2n\sin(n\psi)\cos\psi - (n^2+1) \times\cos(n\psi)\sin\psi]. \quad (17)$$

It is easy to see that, dividing Eq. (17) by Eq. (16), we can eliminate the parameter l from these equations. As a result, we obtain the equation for ψ at the point of the curve self-intersection,

$$\tan(n\psi)[(n^2-1)\tan^2\psi - 2] + 2n\tan\psi = 0.$$

With the help of de Moivre's formula, this trigonometrical equation can be brought to the algebraic form

$$[(n^2-1)h^2 - 2]\mathrm{Im}(1+ih)^n + 2nh\,\mathrm{Re}(1+ih)^n = 0, \quad (18)$$

where we put $h=\tan\psi$. Solving this equation of the nth order, we can define the value of h and, consequently, of ψ for any n. At given ψ, the sought-for critical values of the parameter l can be determined from the relation (16), which can be transformed into the quadratic equation with respect to the unknown quantity l_n.

For $n=3$, the expression (18) transforms into the trivial equation $h^2=1$, from which it follows that the self-intersection takes place at $\psi=\pi/4$. For this value of the parameter ψ, the condition (16) reduces to solving the equation

$$17l_3^2 - 18l_3 - 63 = 0.$$

Its root satisfying the condition $1<l_3<3$ is $l_3=(9+24\sqrt{2})/17 \approx 2.52$. If $l=l_3$, the volume occupied by liquid loses its simple connectivity and one jet splits into four unequal parts. This situation is illustrated in Fig. 3.

For $n=4$ (see Fig. 4), we obtain from Eq. (18) $h^2=5/13$, and Eq. (16) transforms into the equation

$$443l_4^2 - 486l_4 - 2957 = 0.$$

It follows herefrom that $l_4=(243+370\sqrt{10})/443 \approx 3.19$. For the next azimuthal number, $n=5$, we deduce from Eq. (18)

$$3h^4 - 24h^2 + 5 = 0,$$

and hence $h^2 = 4 \pm \sqrt{43/3}$. Substituting this value of the parameter h into Eq. (16), we get $l_5 \approx 3.85$. In a similar manner, we can find that $l_6 \approx 4.55$ and $l_8 \approx 5.85$.

Thus, we have defined the values of the parameter l for which the solutions of the problem of the equilibrium configurations of the jet surface exist. Let us now consider a jet with given characteristics (the cross-section area S and the surface tension coefficient α) and determine the linear charge densities Q, which correspond to the allowable values of l, i.e., to the interval $l_n \leqslant l \leqslant n$.

For the family of the solutions (13) and (14), the area S can be easily found with the help of the Green's formula (compare with Ref. [11]),

$$S = -\frac{n}{2}\,\mathrm{Im}\int_0^{2\pi/n} z\frac{d\bar{z}}{d\psi}d\psi = \frac{4\pi[(l+1)^2-4n]}{(l^2-1)^2},$$

where the dependence (12) of the complex variable z on ψ has been used. Returning to initial dimensional quantities, we get

$$S = \frac{Q^4[(l+1)^2-4n]}{\pi\alpha^2(l^2-1)^2}.$$

Solving this relation with respect to Q, we arrive at the dependence of the linear charge density on the surface tension α, the jet cross-section area S, and the steady-state solution parameters l and n,

$$Q = \left[\frac{\pi\alpha^2 S(l^2-1)^2}{(l+1)^2-4n}\right]^{1/4}. \quad (19)$$

It enables us to determine the critical values of the charge density.

In the problem under consideration, we can define two critical charge-density values for every azimuthal wave number n. The first critical density value Q_n corresponds to the threshold of linear instability of the jet with a round cross section. It can be calculated from the formula (19), where we must take $l=n$ [recall that for $l=n$ the expressions (13) and (14) define circles],

$$Q_n = [\pi(n+1)^2\alpha^2 S]^{1/4}.$$

The critical charge density is seen to grow monotonically with n. Its minimum value corresponds to $n=2$,

$$Q_2 = (9\pi\alpha^2 S)^{1/4} \approx 2.31\alpha^{1/2}S^{1/4}.$$

If the linear charge density Q of the jet exceeds this value, an electrohydrodynamic instability of the jet surface will develop. At the initial stage of the process, this leads to the elliptic deformation of the jet cross section. The disturbances corresponding to the modes with lesser spatial scales, $n>2$, can grow only at larger values of the linear charge density.

The second critical density value $\tilde{Q}_n$ corresponds to the situation when the volume occupied by the jet loses its simple connectivity and the jet splits into two or more separate jets (see Figs. 2–4). We can find it substituting $l=l_n$ into Eq. (19),

$$\tilde{Q}_n = \left[\frac{\pi\alpha^2 S(l_n^2-1)^2}{(l_n+1)^2-4n} \right]^{1/4}.$$

The charge density takes the minimal value for $n=2$, whence it follows that the jet splitting into two approximately equal parts can be considered as the most probable scenario of the jet disintegration. Since $l_2=13/7$, the exact minimal value of the second critical density is given by the following expression:

$$\tilde{Q}_2 = (1800\pi\alpha^2 S/49)^{1/4} \approx 3.28\alpha^{1/2}S^{1/4}.$$

Note that the critical densities Q_n can be found from the linear analysis of the stability of the round jet surface (see, for example, Refs. [3,12]). In so doing, it is not necessary to know the exact solutions for the stationary jet shape. However, we must know them for determining the conditions of the jet splitting or, in other words, for finding the second critical charge densities.

V. STABILITY ANALYSIS

Now consider the dependence of the jet surface deformation amplitude on the linear electric charge density. Such an analysis will allow us to draw some qualitative conclusions concerning the stability of the solutions obtained.

It is convenient to take Q_2 as a unit of linear electric charge density and the radius of the unperturbed jet $R_0 = \sqrt{S/\pi}$ as a unit of length. This implies introducing both dimensionless charge density q and the deformation amplitude of the jet surface r,

$$q(n,l) \equiv \frac{Q}{Q_2} = \left[\frac{(l^2-1)^2}{9[(l+1)^2-4n]} \right]^{1/4},$$

$$r(n,l) \equiv \frac{R_{\max}-R_0}{R_0} = \frac{2\sqrt{n^2-l^2}+(l-1)\sqrt{n^2-1}}{\sqrt{(n^2-1)[(l+1)^2-4n]}} - 1,$$

where $R_{\max}= y|_{\psi=0}$ is the maximum distance between the jet axis and its surface. These relations give the dependence of r on q for different n in the parametric form (l plays the role of the parameter). The relevant plots are presented in Fig. 5. The straight line $r=0$ in the figure corresponds to the unperturbed state of the system, i.e., to the round jet.

One can notice that the deformation amplitude monotonically increases with the charge density for $n=2,3,4$. This

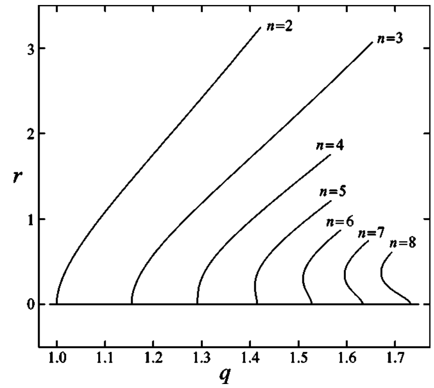

FIG. 5. The dimensionless amplitude of the jet surface deformation r as a function of the normalized linear charge density q for different azimuthal numbers n. The straight line $r=0$ corresponds to the unperturbed state of the system, i.e., to the round jet.

situation corresponds to the soft loss of stability of the round jet. Indeed, it is clear that if $q<Q/Q_n$, then the round jet is stable with respect to small surface perturbations with azimuthal wave numbers greater than or equal to n. Bifurcations occur when the condition $q=Q/Q_n$ holds (supercritical bifurcation for $n=2,3,4$). It is seen from the figure that the side branches corresponding to our exact solutions fork from the straight line $r=0$. It should be noted that the authors of Ref. [11], in terms of the present paper, have plotted the dependence of q^2 on p that also indicates the manner in which the solution branches bifurcate.

As the trivial solution $r=0$ is unstable for $q>1$, the free energy of the system can have a minimum on these branches only. This suggests that the surface modes with small azimuthal wave numbers are excited in a soft regime and, at least for a small overcriticality, our exact solutions can be stable with respect to small perturbations that do not violate the problem symmetry.

For $n>4$, the surface deformation amplitude r decreases with an increase in q in the vicinity of the branch points (see Fig. 5). Such a dependence of the amplitude r on the control parameter q corresponds to the subcritical bifurcation. Then the potential energy of the system, i.e., the sum of the surface and electric field energies, has a maximum on the side branches. It immediately follows that our solutions having azimuthal mode numbers $n>4$ are unstable with respect to small changes in the amplitude r.

Since both the first and second critical charge densities are minimal for the large-scale azimuthal mode $n=2$, it is the surface mode that will define the jet behavior. If the linear charge density Q exceeds the value Q_2, then the surface of the cylindrical jet with a round cross section becomes unstable. As the instability regime is soft, then, for a small subcriticality, a new stable state corresponding to our stationary solutions with $n=2$ appears, and the round jet transforms to an elliptic one. Unfortunately, the analysis of the balance conditions for the forces acting on the jet surface cannot provide the answer to the question as to whether our solutions are stable for all permissible values of the electric charge density, $Q_2<Q<\tilde{Q}_2$ (we discuss the stability with respect to surface perturbations that do not violate the problem symmetry). In all probability, they are unstable for a sufficiently large Q. The point is that $Q_3 < \widetilde{Q}_2$ and $Q_4 < \widetilde{Q}_2$, and, consequently, the functional of the jet potential energy can have several extremums corresponding to different n at given Q. Moreover, as evident from Fig. 2, the jet cannot be stable if the charge-density value is close to $\widetilde{Q}_2$. An arbitrary small deformation of the surface can lead to the rupture of the neck. Thus the condition $Q > \widetilde{Q}_2$ may be considered as the sufficient condition for splitting the jet into two.

VI. CONCLUDING REMARKS

In the present work, we have obtained the two-parameter family of the exact solutions of the classical problem in electrostatics, namely the problem of finding the equilibrium configuration of a charged jet of a conducting liquid. The approach applied is based on the conformal mapping of the region outside the jet to the half-plane, which has restricted our consideration to the case of the plane symmetry of the problem, when all quantities depend only on two variables x and y (see Fig. 1). Because of this, all the solutions obtained correspond to the azimuthal deformations of the jet surface, whereas the deformations in the direction of the jet axis have not been considered. At the same time, it is just the longitudinal instabilities (the varicose and sinuous modes) that determine the behavior of a charged jet in the general case [3]. Nevertheless, if the electrohydrodynamic instability of the liquid cylinder is suppressed in the direction of the z axis, our solutions can play a dominant role in the jet behavior. For instance, the growth of the axial disturbances can be stabilized by the magnetic field directed along the jet axis. It is known [2] that the tangential magnetic field retards the development of the surface instabilities which bend the field lines (in particular, this phenomenon is used to confine the plasma). If the magnetic field is sufficiently strong, the only azimuthal instability of the jet surface will develop. As was discussed in the previous two sections, such instability leads to the jet splitting.

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